A GRAPHICAL METHOD OF DETERMINING THE AVERAGE INCLINATION OF A LAND SURFACE FROM A CONTOUR MAP

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One of the most significant elements of geographic environment is the slope or inclination of the land surface. On it depends, in a large measure, the rate of run-off of the precipitation; the rate of erosion; indirectly, the character of the soil; and finally, the possible utilization of the land for agriculture. In connection with problems of water supply and flood control, a determination of the average inclination of the land surface is of vital importance to the engineer.

In geographical descriptions of a region we seldom find any quantitative mention of the inclination of the surface. The more recent descriptions commonly state the stage in the erosion cycle, and some of them give an indication of the average relief, but these are not sufficient, in all cases, to convey a definite idea of the topography. The description of such a region would, obviously, be improved by adding to the discussion of its stage in the erosion cycle and to the mention of its average relief a statement of the average inclination of its surface.

By the average inclination of the surface of a region is meant an average in which the inclination of each individual portion is weighted in proportion to the ratio which its area bears to that of the whole. Stated mathematically, the formula[1] is:

$$B = \frac{B_1 g_1}{G} + \frac{B_2 g_2}{G} \ldots + \frac{B\ g_n}{G}$$

where B is the average inclination of the region; B_1, B_2, etc., the average inclination of the individual portions; g_1, g_2, the areas of the individual portions; and G the whole area.

The principle heretofore used for obtaining such a weighted average is that published in 1890 by Finsterwalder.[2] This writer shows that the sum of the lengths of the contour lines on a map multiplied by the contour interval, reduced to like units, and divided by the area gives a result which is as near

[1]Penck, A., "Morphologie der Erdoberflaeche," I. Buch, s 47. Stuttgart, 1894.
[2]Finsterwalder, S., "Ueber den mittleren Boschungswinkel und das wahre Areal einer topographischen Flaeche." Sitzber. K. Ak. der Wiss., Math.-Phys. Kl., XX, 1890, s 35-82.

to the true value of the average inclination as can be obtained. The accuracy of the result depends on the contour interval, on the faithfulness with which the topography is represented on the map, and on the accuracy of the measurements of the lengths of the contours and of the area.

Finsterwalder's method is doubtless the best that can be devised for maps on which the contours are few, far apart, and moderately smooth, but it is very laborious and subject to considerable inaccuracy of measurement when applied to maps on which the contours are closely spaced.[3] There are many occasions when a less laborious method, even if less accurate, is desirable, and when the graphic method here proposed should prove useful.

The graphic method is based upon the principle, demonstrated by Finsterwalder[4] that the tangent of the angle of the weighted average inclination of a profile line is the arithmetrical sum of its acclivities and declivities divided by its projected length.

In accordance with this principle, the tangent of the angle of average (weighted) inclination of a meandering profile drawn across a map would be equal to the sum of all the acclivities and declivities divided by the projected length of the profile. Such a profile, if drawn across a map in such a way as to be at all points at right angles to the strike of the surface, (crossing all contours at right angles) would permit the determination of the actual average inclination of the land surface along the line of the profile. A network of these meandering profiles, all drawn at right angles to the strike of the surface at every point, furnishes a means of determining the average inclination of the surface of the region with a degree of accuracy which, as will be shown in the following paragraphs, depends on the nature of the topography and on the number and locations of the profiles.

In practice it is not necessary actually to draw the profiles. The operation is carried out graphically as follows: with a straight edge of paper and a sharp, hard pencil, begin at some point chosen at random in the area to be measured. Lay the

[3]A map-measure, a self-recording toothed wheel so constructed as to turn freely on a pivot, is on the market and will be found very useful for making linear measurements of meandering lines such as contours.

[4]Loc. cit.

paper edge perpendicular to the contours, and, by noting the contour intersections, count and add together the differences in elevation, both positive and negative, which one would encounter in traveling along the line of the paper edge. Continue thus to a point where the contours change direction. Without removing the paper, pivot on the pencil point and turn the paper until it is once more perpendicular to the contours. Proceed as before. Continue thus until the limits of the area to be measured are reached, always keeping the paper edge at right angles to the contour lines and counting and adding arithmetically all differences in elevation, whether positive or negative. Choose another point and repeat the process, continuing to add distances graphically along the paper edge, and to record the total "ups" and "downs" of the profiles. When the map has been sufficiently covered, scale off the total distance represented along the paper edge, find the sum of all "ups" and "downs," and divide the latter by the distance. The result is the tangent of the angle of average inclination of the surface. As a concrete example, suppose that the total length of the profiles is 10 miles and the sum of the ups and downs is 6630 feet, then $\frac{6630}{10x5290}$ =tan. angle of inclination, or, stated in another way, an inclination of 6630 feet in 10 miles$=\frac{6630}{10}=$ 663 feet per horizontal mile.

To compute graphically the angle of inclination, express both quantities in the same units; plot the sum of the ups and downs as a perpendicular line at one end of the line representing the length of the profiles; draw the hypothenuse, and measure the angle with a protractor.

The accuracy of the results obtainable by the proposed graphic method obviously depends on (a) the character of the topography, and (b) the completeness with which the various topographic units in the region find weighted expression in the profiles. In general, the closer the net of profiles, the more accurate the result. On topography with moderately uniform slopes, such, for instance, as that of a maturely dissected plateau, a very close approximation of the average inclination of the surface may be obtained in a few moments from a very few random lines. On topography of very uneven character, some artificial device must be adopted which insures that each topographic unit shall be weighted in proportion to its area. To a certain extent this may be done by judgment alone, but some empirical method which, so far as possible, eliminates the personal equation is preferable. A method which gives good results and is here recommended, is to divide the map into large or small squares of equal area, and, by the ordinary method, run as many random lines across each square as is necessary to yield a reliable average of the inclination of its surface. Add all distances graphically along the paper edge and obtain the sum of the ups and downs in the usual way. It is essential that the same number of lines be run in each square, otherwise those in which the most lines are run receive the greatest weight in the average. This method should yield results with an error of 5 per cent or less, the accuracy depending on the care with which the work is done and the size of the squares used as units. The accuracy thus obtainable compares favorably with that obtained by Finsterwalders' method of measuring the lengths of the contours.

In judging the character of the net of profiles necessary to obtain the average slope of an area it should be borne in mind that a single measurement of a truly conical hill, or of one in which the slopes are uniform, gives a correct average, but that on a hill or valley head of circular form which has a parabolic profile, with the steeper slope at either top or bottom, the results are in error on account of the inequality of the areas having the various degrees of slope.

On most examples of mature or old topography the majority of the slopes are parabolic, but since, in general, the valley heads are the inverse of the spurs, the errors due to the parabolic curve of the surface profiles tend to balance each other where both valley heads and spurs are included in the measurements.

For certain determinations, however, such as that of the average inclination of a conical mountain, e. g., a volcano, having a parabolic profile, a special method of compensation must be employed. This may be accomplished by drawing concentric circles round the center of the mountain with radii of 1, 2, 3, 4, 5, 6, 7, 8, etc., and increasing the number of measurements in each succeeding belt in proportion to its area, namely, (if the area of the first belt be taken as one) for the first belt, one measurement; for the second, three; for the third, five; the fourth, seven; etc., in arithmetrical progression. Thus an approximately correct average may be obtained.

There are certain types of topography for which a statement of the average inclination has little meaning from the geographer's standpoint, though it still retains its value to the engineer. Such are the types in which large and distinct units of area possess markedly different degrees of inclination, while each by itself is relatively uniform. The till plains of central Illinois are examples. Broad, flat uplands are cut by narrow, steep-sided valleys. A statement of the average inclination of such a surface has small geographical value. What is needed is a distinction of the two areas, and a statement of the average inclination of each, together with an indication of its area.

For the determination of the average inclination of each division of such a region, the graphic method is admirably suited, while Finsterwalder's method of measuring the lengths of contour lines is, depending on the specific conditions, either inapplicable or very complicated.

SUMMARY

In connection with several lines of investigation a statement of the average inclination of a land surface is desirable. The best method formerly used is, in many instances, slow and extremely laborious, and is subject to considerable errors of measurement. The graphical method here proposed is simple and easily and quickly applied. Its accuracy depends somewhat on the character of the topography, and is, to a large degree, proportional to the number of measurements taken. It, therefore, serves either for quick, rough determinations, or for those in which the maximum accuracy is required. By the use of a close network of small squares the method may be made to compare favorably in accuracy with its competitors at a considerably lower labor cost.