WIND STRESSES IN STEEL SKELETON CONSTRUCTION

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I. DEFINITION OF STEEL SKELETON CONSTRUCTION

By steel skeleton construction is meant that type of building construction in which the frame is made up of a system of steel columns and girders so planned that the loads from the walls and floors are delivered by the girders to the columns at each story. Each floor slab is supported on steel beams, and at each floor the walls are supported on steel beams framed into the columns, so that the floors and walls of one story, so far as support is concerned, are entirely independent of the floors and walls of all other stories. As seen in elevation, the steel work of such a building is made up of a series of rectangular frames. This is the type of construction that is used in practically all modern high buildings.

II. HOW A STEEL SKELETON FRAME RESISTS WIND PRESSURE

The wind pressure on a building acts upon the outside wall and is delivered by the wall to the steel frame. It is customary to assume that the wind load is applied to the steel frame at the level of the floors only. A bent of a building which is designed to resist the wind load is, therefore, considered as being acted upon by a series of concentrated horizontal forces at the various floors. The force at the top of the top story tends to make the top of the story more horizontally relative to the bottom of the story. For a twenty-story bent, the force at the top of the twentieth story and also the one at the top of the nineteenth story, tend to make the top of the nineteenth story move horizontally relative to the bottom of that story. In the same way, for any story, all of the forces above the story tend to make the top of the story in question move horizontally relative to the bottom of that story. The bottom of a story is prevented from moving horizontally because of its connection to the foundation by means of the intervening stories of the bent. For any story, therefore, the forces acting on the bent above the story in question tend to move the top of the story horizontally in one direction, whereas the stories below act horizontally upon the bottom of the story, in the opposite direction, to prevent it from moving horizontally in response to the forces above. That is, there is a shear in the story, and this shear is equal to the sum of the horizontal forces acting upon the bent above the story in question, including the force at the top of the story.

Fig. 1 represents the tenth story of a three-span bent. The force W applied at the eleventh floor is equal to the sum of all the horizontal forces above, including the one at the eleventh floor. W tends to push the top of the tenth story to the right. The force w is the wind load at the tenth floor. The

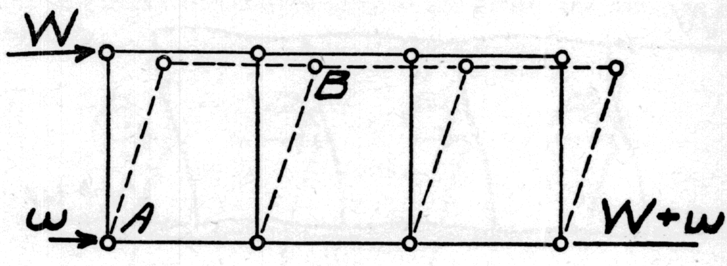

FIG. 1

force W+w is the resistence which the ninth story offers to prevent the bottom of the tenth story from moving to the right. If the columns and girders were hinged at the corners the frame would collapse as indicated by the dotted lines. The bracing added to prevent the frame from collapsing is called the wind bracing.

The easiest way to prevent the frame from collapsing would be to put in a diagonal tension member from A to B. This, however, it is not practicable to do inasmuch as diagonal bracing in the exterior walls would interfere with the windows, and all interior walls must be made so that they can be removed to meet the changing needs of the tenants. It is therefore necessary to make the frame capable of resisting shear without interfering with the clear rectangular space between the columns and girders.

If the connections between the columns and girders are made rigid, that is, if the columns can not turn relative to the girders, the frame, when subjected to a shear, instead of collapsing will take the form shown in Fig. 2. The columns of this latter frame tend to fall over to the right, the same as the columns of the frame in Fig. 1, but the girders at the tops of the columns hold the top ends in a nearly vertical position and likewise the girders at the bottom hold the bottom ends in a vertical position. That is, a column as a whole is not free to fall over, and the top can only move to the right by moving relative to the bottom when both the top and bottom remain in a nearly vertical position. Under these conditions a column is capable of resisting a horizontal shear.

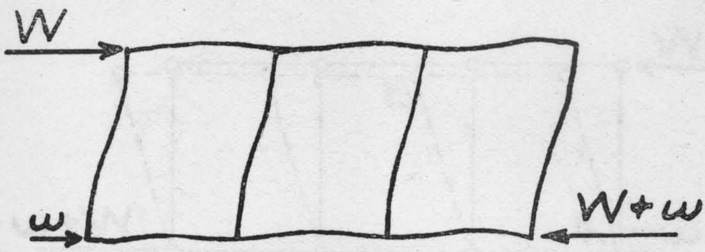

FIG. 2

The columns naturally take the position shown by the dotted lines in Fig. 1, that is, they tend to rotate and they are prevented from rotating only because the girders hold the ends in a nearly vertical position. In other words, and this is a fact that should be kept clearly in mind, the columns are the overturning members and the girders are the resisting members.

As stated above, the columns represented in Fig. 2 are subjected to shear. Since it is considered that no horizontal forces act upon the steel frame between the floors, the shear on a column is uniform for a story height. This shear produces a bending moment in the column, that is, the stress in the column due to the wind is tension on one side and compression on the other. From the form which the column takes when strained, it is apparent that for a particular side of any column if the stress at one end is compression that at the other end is tension, or the reverse. That is, the bending moment and also the bending stress changes sign, or passes through zero, some place between the two ends of the column.

The point where the moment changes sign is called the point of contra-flexure. The conditions at this point are the same, as far as stresses are concerned, as if the column were cut and the two parts were connected with a frictionless hinge. There is a shear but there is no moment at the point of contra-flexure.

Since a girder is subjected to forces resulting from the wind load only at points where the girder is connected to the columns, the shear on a girder is uniform between columns. The shear in the girder produces a bending moment and this bending moment changes sign in the girder the same as in a column.

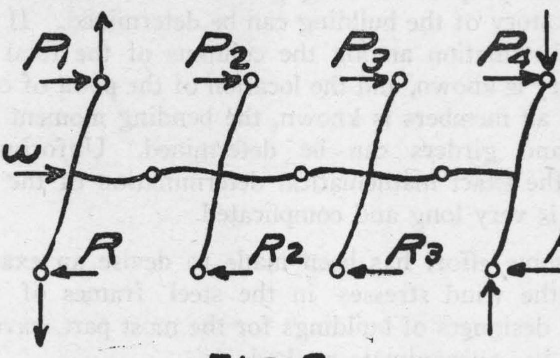

FIG. 3.

III. APPROXIMATE METHODS OF CALCULATING WIND STRESSES

In Fig. 2 the action of the portion of the bent above the tenth floor, upon the tenth floor is represented as a single force W; and the action of the tenth story upon the story below is likewise represented as a single force W+w. In reality the horizontal shear is transmitted from story to story through the columns.

If a column is divided at the point of contra-flexure, represented in Fig. 3 by a small circle, the upper portion of the column will exert upon the lower portion a horizontal shear, but no moment. (There is also a vertical force, but it is neglected in this discussion.) The total shear above the tenth floor is represented by W. It is equal to the sum of the shears, P_1, P_2, P_3 and P_4, in the columns. The shear is constant between the tenth and eleventh floors, whereas the moment varies and passes through zero at the point of contraflexure.

If the columns in the story just below the tenth floor are divided at the point of contra-flexure the lower portion of each column will exert upon the upper portion a shear, but no moment. These shears are represented by R_1, R_2, R_3 and R_4. Likewise if each girder is divided at its point of contraflexure each part would exert upon the other part a shear, but no moment.

If the dimensions of a building are known and a wind pressure, for the purpose of design, is assumed, the total shear upon each story of the building can be determined. If therefore the distribution among the columns of the total shear upon a story is known, and the location of the point of contraflexure of all members is known, the bending moment in the columns and girders can be determined. Unfortunately, however, the exact mathematical determination of the above quantities is very long and complicated.

While some effort has been made to devise an exact analysis of the wind stresses in the steel frames of office buildings,[1] designers of buildings for the most part, have been content to use approximate methods.

Four approximate methods have been used. For convenience in reference these will be designated as Method I, Method II, Method II and Method IV, respectively. Mr. R. Fleming presented the first three methods in Engineering News.[2] These methods, as applied to a building in which all columns of a story have the same section, are based upon the following assumptions:

ASSUMPTIONS IN METHOD I

1. A bent of a frame acts as a cantilever.

2. The point of contra-flexure of each column is at mid-height of the story.

[1]Wind Stresses in the Frames of Office Buildings, by Albert Smith, Journal Western Society of Engineers, Vol. XX, No. 4, p. 341.
Stresses in Tall Buildings, by Cyrus A. Melick, Bulletin No. 8, College of Engineering, University of Ohio.
The Theory of Frameworks with Rectangular Panels and Its Application to Buildings which have to Resist Wind, by Ernst F. Jonson, Tran. Am. Soc. C. E., Vol. 55, p. 413.

[2]Wind Bracing Without Diagonals for Steel-Frame Office Buildings, Engineering News, March 13, 1913.

3. The point of contra-flexure of each girder is at its mid-length.

4. The direct stress in a column is directly proportional to the distance from the column to the neutral axis of the bent.

ASSUMPTIONS IN METHOD II

1. A bent of a frame acts as a series of portals.

2. The point of contra-flexure of each column is at mid-height of the story.

3. The shear is the same on all columns of a story.

4. Each pair of adjacent columns of a bent acts as a portal, and each interior column is a member of two adjacent portals. The direct stress in an interior column, when the column is considered as a member of the portal on one side, is of opposite sign from the direct stress in the same column when considered as a member of the portal on the opposite side and the resultant direct stress is equal to zero.

ASSUMPTIONS IN METHOD III

1. A bent of frame acts as a continuous portal.

2. The point of contra-flexure of each column is at mid-height of the story.

3. The direct stress in a column is directly proportional to the distance from the column to the neutral axis of a bent.

4. The shear is the same on all columns of a story.

Professor Albert Smith, in a paper before the Western Society of Engineers, describes a method which he has used in his classes in Structural Engineering at Purdue University. This method is here designated as Method IV:

ASSUMPTIONS IN METHOD IV

1. The point of contra-flexure of each column is at mid-height of the story.

2. The point of contra-flexure of each girder is at its mid-length.

3. The shears on the internal columns are equal and the shear on each external column is equal to one-half of the shear on an interior column.

If all of the assumptions of any one of these methods are accepted, the stresses in a frame may be determined by applying the fundamental equations of static equilibrium.

4. *Slope-Deflection Method.* — The four approximate methods described above are all short and simple and, when any one of them is considered by itself, the assumptions upon which it is based seem reasonable. It is an unfortunate fact, however, that if the four methods are applied to the same frame, the results obtained by the four methods are radically different. Furthermore in the absence of an accurate method to be used as a standard of comparison, it was impossible to judge of the relative accuracy of the approximate methods.

The writer, assisted by Mr. G. A. Maney, devised a method known as the slope-deflection method to be used to determine the relative accuracy of the approximate method.*

It can be proven that the moment at the end of a member in flexure is given by the equation

$M_{AB}=2\ E\frac{L}{I}(2\ \Theta_A+\Theta_B-3\frac{d}{L})$, in which

M_{AB}=the amount at A in the member A B.

Θ_A=rotation of the tangent to the elastic curve of member. at A due to the stress, or the change in the slope of the member at the end A.

Θ_B=rotation of the tangent to the elastic curve of the member at B due to the stress, or the change in slope of the member at the end B.

d=deflection of one end of the member relative to the other end, measured in a direction normal to the original position of the member.

L=length of the member.

1=moment of inertia of the section of the member.

E=modulus of elasticity of the material.

*Bulletin No. 80, Engineering Experiment Station, University of Illinois.

That is, the moment at the end of a member can be expressed in terms of the changes in the slopes at the ends of the member and in terms of the deflection of one end of the member relative to the other end.

Neglecting the change in length of a member due to axial stress, the deflections of all columns in a story of a bent are equal, and the deflection of the girders are equal to zero. If the connections between the columns and girders are perfectly rigid all members intersecting in a point are subjected to the same angular strain at that point. The unknown quantities for each story of a bent are therefore one change in slope, or Θ, for each intersection of a column with a girder, and one deflection for the story as a whole. That is, there are as many unknowns per story as there are columns plus one.

If the point in which a girder intersects a column is considered by itself since it is in equilibrium the algebraic sum of the moments acting upon the points equal zero. A moment equation therefore can be written for each intersection of a column with a girder, or as many equations can be written for each story as there are columns in the bent.

If all of the columns in a story of a bent are considered together, the algebraic sum of the moments at the tops and bottoms of all of the columns equals the total sheer in the story multiplied by the story height.

All of the above moments are moments at the ends of the girders and columns. As explained above these moments can be expressed in terms of the changes in slopes, at the ends of the members and the deflections of one end of a member relative to the other end. The total number of equations in one story of a bent is therefore equal to the number of columns in the bent plus one. The number of unknowns in the equations for one story of a bent is equal to the number of equations for the story, but the equations for one story contain unknowns from the story above and from the story below the one in question. It is therefore impossible to isolate the equations for one story and determine the unknowns. There is no story to contain unknowns above the top story, and the bottoms of the columns of the bottom story are usually assumed to be fixed. Under these conditions there are as many equations for a bent as a whole as there are unknowns and the unknowns can be determined.

V. CONCLUSIONS

The number of equations involved in the slope-deflection method is so great that the method can not be used in designing buildings. The method, however, was used by the writer and Mr. Maney to determine the stresses in a number of typical bents, after which the stresses were also determined in the same bents by the four approximate methods described above. As a result of these calculations it was found that for bents for which all columns of a story have the same section Method I and Method IV are accurate enough for the purpose of design.