PSYCHOLOGY OF ARITHMETIC IN THE LIGHT OF RECENT EXPERIMENTATION

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A large number of experiments have been conducted in recent years in the field of arithmetic. In 1925 Buswell and Judd published a monograph containing a summary of all scientific investigations relating to arithmetic which had appeared up to that time.[1] This summary has been kept up to date by the publication each year of a supplement to the monograph—the monograph, together with its supplements, containing a total of 584 titles. In the short time which is at my disposal it is obviously impossible to deal with all of these studies. This paper is restricted to a small number of experiments which deal with the fundamental operations, especially addition and subtraction. An attempt is made to give a brief summary of these investigations and to interpret them in terms of the psychology of arithmetic.

Since 1913 a number of studies have been made to determine the relative difficulty of the various number combinations. These studies used a variety of methods of determining difficulty, some of them employing the number of errors made by large groups of pupils in giving a combination, others measuring the time required by pupils to give the answer for each combination, and still others determining the difficulty of learning the combination. Probably the most outstanding of all of these studies is the one reported by Clapp[2] in 1924, using the first of the methods just mentioned.

Although some of these investigations do not agree very closely as to the degree of difficulty of the various combinations, they all show that some of the combinations are much more, or much less, difficult than others, and especially that the difficulty of a given combination is not necessarily the same as that of its reversal. For example, Clapp's study showed that 3 plus 8 was much more difficult than 8 plus 3, and that 5 times 6 was more difficult than 6 times 5. Similar relationships were found in the case of subtraction and division. The combination 11 minus 9 was much more difficult than 11 minus 2, and 56 divided by 8 more difficult than 56 divided by 7. The studies show that there

[1] Buswell, Guy T., and Judd, Charles H., Summary of Educational Investigations Relating to Arithmetic, Supplementary Educational Monographs, 1925.

[2] Clapp, Frank L., The Number Combinations, Bureau of Educational Research Bulletin, No. 1 and No. 2, University of Wisconsin, 1924.

is some tendency for direct and reverse combinations to resemble each other in difficulty, but they are by no means identical. In other words, a child may know a number fact in one direction but not in the reverse direction.

The results of these investigations upon the relative difficulty of the number combinations have been used by some educators to explain the psychology of arithmetic in terms of the bond theory of learning. It was inferred that each combination had to be reckoned with as a separate entity and taught as such; each was regarded as a bond which had little or no connection psychologically with any other; that is, the learning of one number fact had little or no facilitating influence upon the learning of another.

This interpretation has had a definite influence upon the construction of textbooks and upon methods of teaching. When each of the digits between 0 and 9 inclusive is added to itself and to each of the remaining nine digits, a total of 100 addition facts results. The early writers of textbooks assumed that it was unnecessary to teach the combinations involving zero, and that a child who knew a combination in one direction also knew the reversal. Consequently they provided for the teaching of only 45 addition facts. Similar assumptions and practices were followed in the case of the other fundamental processes. The line of investigation already referred to, however, led to the assumption that each number fact had to be learned separately and specifically. As a consequence modern textbooks provide for the teaching of all of the combinations. The earlier psychology assumed that it was unnecessary to teach certain number facts because of their relationship to others which were taught; the later view assumed that in the child's mind, at least, there were no relationships among the various number facts, and that, therefore all of them would have to be taught, and that they were to be taught as if no inter-relationships existed or could be supplied.

It is this later view which we shall now attempt to examine in the light of recent experimentation. Is it true that from the point of view of the child there are no relationships among the number facts? The results of three experiments which bear upon this question will be summarized at this point.

The first study to be referred to was reported by Buckingham in 1927.[3] In each of seven schools he divided the second grade pupils into two equal groups. One group was taught a list of nineteen addition facts during one week, and the corresponding nineteen subtraction facts during the following week; the second group was taught the same thirty-eight facts, but together rather than separately. Further-

[3] Buckingham, B. R., Teaching Addition and Subtraction Facts Together and Separately, Educ. Res. Bull. 6, pp. 228-229, 240-242, 1927.

more, for the second group the facts taught at any one time were selected so that they were related anl constituted a set. One such set, for example, included the following combinations: 1 plus 6, 6 plus 1, 7 minus 1, and 7 minus 6. The same amount of time and the same methods of teaching were used with both groups of pupils. At the end of the experiment a test over the thirty-eight combinations was given to both groups. In six of the seven schools the results on this test were very strongly in favor of teaching the combinations together rather than separately. In the seventh school the procedure followed by the teacher was so irregular that the results should hardly be considered.

The second experiment was reported in 1930 by Beito and Brueckner.[4] They used a total of 93 second-grade pupils found in three schools, and employed only those addition combinations which are reversible. They had the teachers spend three weeks teaching the direct combinations only, not their reversals. Three different methods of teaching the addition facts were employed. The pupils were tested as to their knowledge of both the direct combinations and their reversals at the beginning and again at the end of the period of instruction. In this way it was possible to determine what number facts a pupil learned during the experiment. The results of the tests showed that the pupils gained slightly more on the reverse combinations, which had not been taught, than on the direct combinations, which had been taught. The experimenters stated, "Although the reverse combinations were never mentioned during the study except in the pretest and the final test, the pupils in some manner made the adjustment to the reverse combinations to such an extent that a very large percentage was learned in the reverse combinations. This result was consistent throughout all classes and for the different methods of presenting the combinations."

The final experiment which is of interest in this connection was reported by Olander in 1930.[5] He employed a total of 37 sections of second-grade pupils taught by 26 different teachers, and continued the investigation over a period of 17 weeks. During the first 11 weeks the pupils were taught a list of 55 selected combinations in addition and an equal number in subtraction. In order to understand the selection of the combinations taught we must recall that the 100 addition facts include 45 pairs of reversible combinations. Six of these pairs were taught in both direct and reverse order, six remained entirely untaught, and the remaining ones were taught in either direct order or in reverse

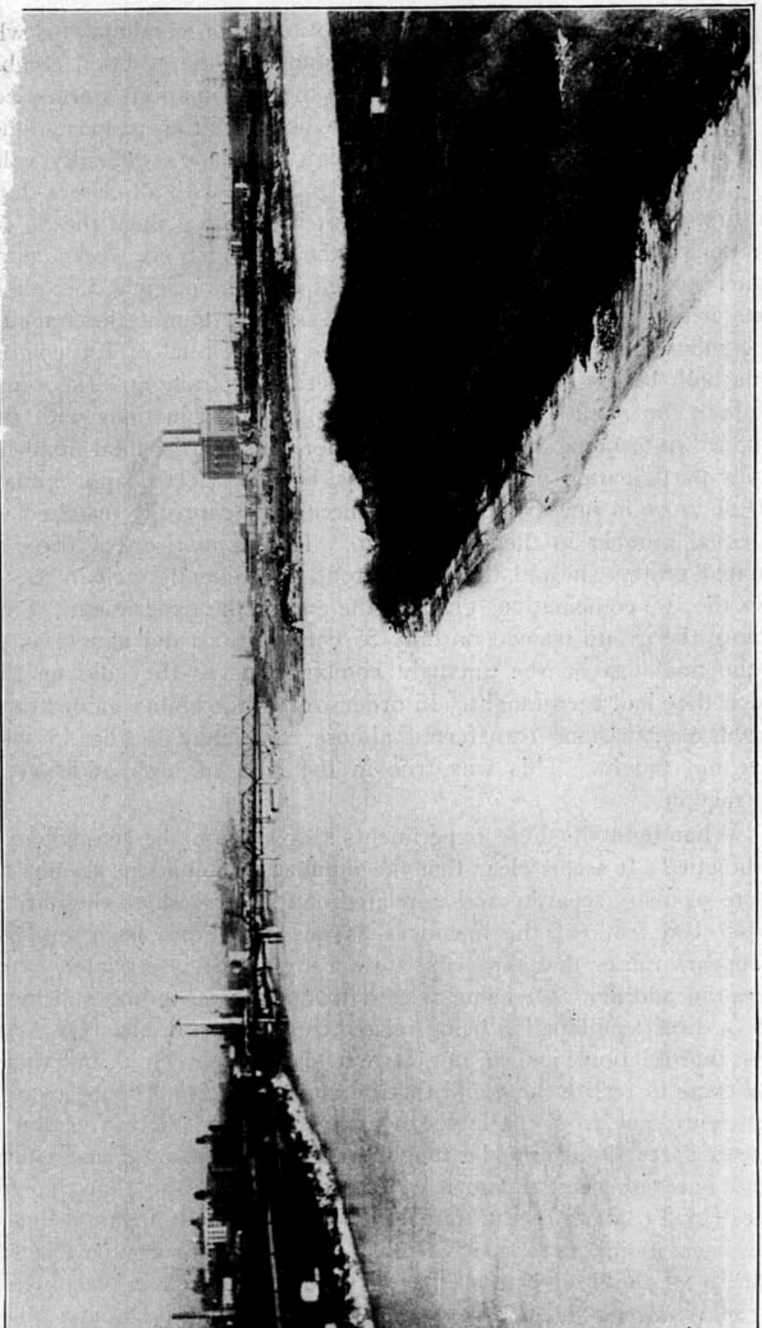
[4] Beito, E. A., and Brueckner, L. J., A Measurement of Transfer in the Learning of the Number Combinations. Twenty-ninth Yearbook, National Society for the Study of Education, pp. 569-87, 1930.

[5] Olander, H. T., Experimental Determination of Transfers in Addition and Subtraction, Doctor's Thesis, University of Chicago, 1930.

order, but not in both directions. The subtraction combinations which were taught were those which corresponded to the addition combinations. In both addition and subtraction the combinations were selected in such a way that those not taught were as difficult as, or more difficult than, those which were taught, according to Clapp's difficulty values.

The same teaching procedure was followed with all classes during the first eleven weeks of the experiment in teaching them the 55 combinations already referred to. During the remaining six weeks approximately one-half of the classes continued with the original 55 combinations while the other half of the classes were taught the remaining 45 combinations. All classes were tested on the total of 100 combinations both before and after this last period of training. In order to compare the results obtained in teaching 55 combinations with those obtained in teaching 100, Olander did not consider the total number of pupils participating in the experiment, but he selected approximately 300 of those in one group who had been very carefully matched with an equal number in the other group. The comparison of these two equated groups showed that both groups did equally well in the test over the 100 combinations given at the end of the experiment. Furthermore the group trained on only 55 combinations did almost as well in the final test on the untaught combinations as they did on those which they had been taught. In order words, the ability gained on the taught combinations transferred almost completely to the 45 which were not taught. This was true in the case of both addition and subtraction.

What light do these experiments throw upon the psychology of arithmetic? It seems clear that the number combinations do not constitute so many separate and unrelated entities or bonds—they are not simply 100 feats of the memory—as has sometimes been supposed. It appears rather that they constitute a system of interrelated experiences, an addition fact being related to the corresponding subtraction fact, a direct combination being related to its reversal, etc. Of course, these interrelationships are not, at first, clear to the child, but that he may come to realize them and that he may profit from being aware of them is evident from the experiments. It follows, therefore, that the number facts should not be taught as so many separate and isolated items, but rather as a system of interrelated facts so that the pupil himself will come to regard numbers in this systematic fashion. Further experimentation is necessary, of course, in order to determine just what elements of the interrelationships shall be emphasized in teaching, but that relationships and not merely separate facts shall be taught follows unquestionably from the experimental evidence which is already available.



Illinois River at Peoria

(Courtesy Peoria Star)