Flexural Vibrations of Piezoelectric Quartz Bars

J. T. Tykociner and M. N. Woodruff
University of Illinois, Urbana, Illinois

Stabilization of high frequency oscillators by means of vibrating quartz crystals became the most important application of piezoelectricity. All radio broadcasting stations are piezoelectrically controlled oscillators in order to enable simultaneous radio transmission by hundreds of stations and to keep each of them within $\pm$ 50 cycles of the allotted frequency channel. The range of frequencies used in broadcasting extends from 550 to 21,520 KC. There are, however, other applications of oscillators of much lower frequencies in the range from 1 to 100 kc. Very little has been done to extend the methods of stabilization for these low frequencies. The main difficulty consists in the necessity of using very large crystals which are required if the usual longitudinal vibrations are applied. For frequencies below 15 kc quartz crystals of required dimensions are not available. To avoid this difficulty it is suggested to apply flexural vibrations of quartz bars. For this purpose an experimental study was made on twenty-one bars and plates cut of a large quartz crystal. Their length coincident with the electrical axis was from 0.7 to 13 cm., their height coincident with the optical axis was from 0.75 to 1.8 cm. and their thickness varied from 0.15 to 0.27 cm.

Two methods were used for the determination of natural frequencies of these crystals. For frequencies above 15 kc. a variable frequency oscillator driven by a thermionic tube was coupled with an aperiodic circuit. The latter consisted of a coupling inductance, the crystal bar mounted in a four-electrode holder and a copper oxide rectifier output meter. The quartz bar acted as an electro-mechanical resonator whenever the oscillator approached the fixed vibration frequency of the quartz bar. The polarity of the four electrodes placed parallel to the electrical axis of the bar was such that the direction of the electric field across the upper pair of electrodes was opposite to the field direction in the lower pair of electrodes. The piezoelectrically active quartz bar was thus subjected to compression in the upper part whenever tension was produced in the lower part and vice versa in accordance with alternating potentials produced by the oscillator at the two pairs of electrodes.

A pulse observed on the output meter indicated resonance for the particular setting of the variable condenser of the oscillator. For the determination of natural frequencies of flexurally vibrating bars which had a natural frequency below 15 kc. a compensating bridge method proved more sensitive. The crystal was mounted in a holder with two pairs of electrodes placed parallel to the length axis of the crystal and inserted in one arm of the bridge in series with a resistance of the second arm. The third arm consisted of a variable condenser in series with a resistance of the fourth arm. The connection between the two adjacent resistances and that between the crystal holder and the variable condensers were used as points of potential difference at which energy was supplied from an oscillator of variable frequency. The potential difference at the two points where one resistance was connected to the variable condenser and the other resistance to the crystal mounting served to energize the input circuit of an amplifier. The output circuit of the latter was connected to a copper oxide rectifying detecting instrument which similarly, as in the first method, indicated a pulse whenever the frequency of the oscillator was made equal to the natural frequency of the resonating quartz bar. For each quartz bar rebalancing of the bridge was required. Longitudinal vibrations were also measured by a simpler method. The results of measurements confirmed the known relation that the natural frequency of the bars vibrating longitudinally is independent of the height (a) or thickness (t) of the bar and is a linear function of the length (b) of the bar. As to the flexural vibrations it was found that their natural frequencies do not correspond to the known relation $f = A \frac{a}{b^2}$. Frequencies calculated according to this relation gave values from 2 to 75 per cent larger than the measured ones. The deviation depended on the ratio a/b of the crystal. For crystals whose length was over ten times larger than its height, the measured fundamental frequency was in close agreement with the above relation. For crystals with a ratio $a/b = 0.5$ the discrepancy amounted to 40 per cent and for a square plate, $a/b = 1$, the discrepancy reached 60 per cent.

For the development and design of piezoelectrically controlled low frequency oscillators it was important to find precise relations between the natural frequencies and the dimensions of flexurally vibrating quartz crystals. By considering the rotary inertia of each cross-section of a vibrating bar in addition to its energy of translation Rayleigh derived a correction factor which is a function of a/b. Timoshenko extended the basic differential equations by including the effect of shear. The frequencies calculated with Rayleigh's correction were still too high and those calculated on the basis of the equations given by Timoshenko and Goens were too small as compared with the measured frequencies. From the experimental data collected on the twenty-one crystals a satisfactory expression was obtained for the frequency of flexural vibration. The following procedure was used.

Assuming that the relation searched for has the form $f = \frac{A}{B} \frac{a/b}{b}$ where A is a constant and B is a function of a/b. The ratios A/B were calculated for each crystal from the measured frequencies f and the geometric dimensions a and b. These values A/B were then plotted against a/b. For B = 1 the value of the constant $A = 5.52 \times 10^5$ was obtained by extrapolation. Because of the uncertainty of extrapolating, this value which represents 1.03 times the velocity of propagation in the length direction was checked experimentally by measuring for each crystal its natural frequency of longitudinal vibration. The average experimental value was found to be $A = 5.54 \times 10^5$ cm. Dividing each of the values A/B by A, a curve for the relation B as function of a/b was obtained. It was found that it represents closely the relation $B = \sqrt{1 + 5.22\,(a/b)^2}$. Thus the natural frequency for flexural vibrations may be calculated from the expression $f = \dfrac{5.52 \times 10^5\, a}{\sqrt{1 + 5.22(a/b)^2}\, b^2}$ where a and b are the height and length of the crystal measured in cm.

The frequencies calculated from this formula deviate little from measured values. For 15 quartz bars this deviation was less than ± 1%, for 4 crystals it was from ± 1% to ± 1.8% and for two crystal plates which were the smallest (a = 1.05, b = 1.103 and 0.721 cm.) the deviation was — 6.83% and + 4.85% respectively. These latter two crystals with a/b = 0.957 and 1.467 respectively cannot be regarded as bars. They were included in this investigation for the purpose of determining the range of a/b within which the above formula can be applied.

The following conclusions may be made from this investigation:

(1) It is possible to produce flexural vibrations of quartz crystals in a plane containing its two largest dimensions even if the height (a) exceeds the length (b).

(2) A relation was obtained for the calculation of the natural frequencies of flexural vibrations from the dimensions of the crystals.

(3) This relation enables the determination of the dimensions of quartz resonators necessary for piezoelectric control of oscillators within a range of frequencies from about 3 to 120 kc. Its accuracy is better than ± 1.5 per cent for crystals whose ratio a/b does not exceed 0.5.

More details will be published in a bulletin of the University of Illinois Engineering Experiment Station.