

THE USE OF THE PERIDOGRAM IN ESTABLISHING THE REALITY OF HIDDEN OR SUSPECTED PERIODICITIES

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The investigation for the evidence as to the reality of a periodic change in the intensity of cosmic rays with respect to sidereal time is rendered unusually difficult for three reasons:

1. The sidereal diurnal period is only four minutes shorter than the solar diurnal period, or about $1/365$ of a solar day.
2. In addition to this very minute difference in the periods, the sidereal variation is much smaller than the solar variation.
3. The amount of data available is limited and does not extend in most projects beyond three or four years.

The appearance of a sidereal variation upon mathematical analysis by no means establishes the reality of the sidereal variation, for with a seasonal change there is introduced by formal analysis a variation which is spurious at a frequency corresponding to the sidereal day. The appearance of such a spurious frequency further complicates the problem.

If a function $f(t)$ be periodic with two periods T_1 and T_2 , and if

$$a = \frac{2}{Tn} \int_0^{nT} f(t) \cos kt \, dt$$

$$b = \frac{2}{Tn} \int_0^{nT} f(t) \sin kt \, dt$$

$$c^2 = a^2 + b^2$$

then c^2 which depends upon T , the period, n , the number of observations, and the amplitudes of the variations will have values other than zero at points corresponding to the two periods. The values between the points corresponding to the periods T_1 and T_2 may not be zero if the amplitudes are large and the observations few in number. The curve obtained by plotting c^2 against T is called the *perido-gram*. Beyond the abscissae T_1 and T_2 the value of c^2 falls off to zero very rapidly.

The ordinates of the two peaks (at T_1 and T_2) will depend upon the amplitudes of the two variations. Here again, be-

of the finite limits of the Fourier Integrals spurious periodicities are introduced giving secondary maxima on either side of the principal maximum, but these spurious maxima may be distinguished from the real (fig.1a), since with increasing data these secondary peaks move toward their respective principal maxima (fig.1b and 1c).

The calculation of amplitudes corresponding to different periods may be performed by Fourier analysis; however, this involves a great deal of work, and since in the present problem the periods are close, this work can be greatly simplified as outlined in the following paragraphs.

The Fourier analysis was made for the period of one solar day with groups of days averaging about fifteen. Amplitudes

corresponding to periods slightly smaller and larger than one solar day and extending beyond the period equal to the sidereal day may be obtained by reference to the harmonic dial. If the maximum value of the function

$$f(t) = a \cos \frac{2 \Pi t}{T} + b \sin \frac{2 \Pi t}{T} \\ = c \sin \frac{2 \Pi t}{T} + \phi$$

where T is one solar day

is represented on a twenty-four hour clock dial by a vector, its length will indicate the amplitude, and its direction, the time of maximum. On the other hand, if this be done for a cloud of days, then the vector will indicate the mean amplitude and the mean time of maximum.

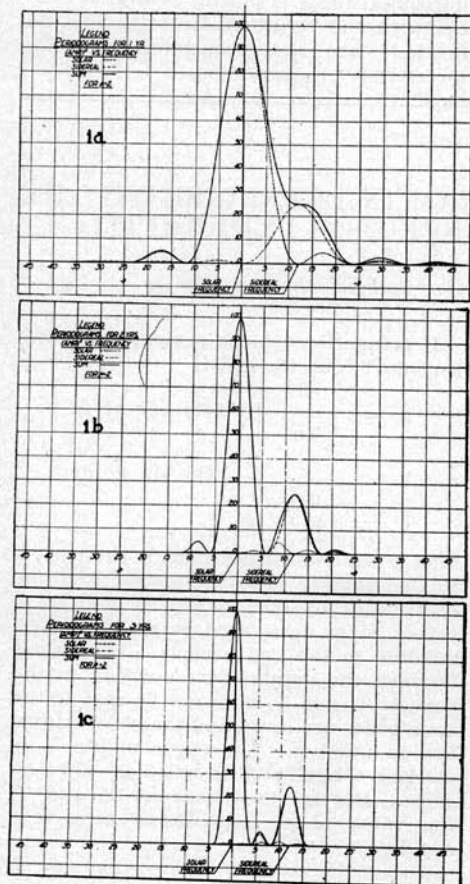
If the function $f(t)$ is to be investigated for a period other than that represented on the face of the harmonic dial, then the vector for each partial cloud must be rotated through an angle depending upon the trial period. The amplitude c corresponding to the trial period T' is given by the following set of equations:

$$a' = \frac{1}{n_s} \sum_{m=1}^s (n_m - n_{m-1}) \left[\begin{array}{l} a_m \cos \alpha_m - \\ b_m \sin \alpha_m \end{array} \right] \\ b' = \frac{1}{n_s} \sum_{m=1}^s (n_m - n_{m-1}) \left[\begin{array}{l} a_m \sin \alpha_m + \\ b_m \cos \alpha_m \end{array} \right] \\ \alpha_m = (n_{m-1} + n_m) \frac{\Pi T - T'}{T}$$

where n_s is the total number of days analyzed ($n_m - n_{m-1}$) is the weighing factor and is the number of days in the m th group, and s is the number of groups.

After a suitable number of trial periods have been calculated the amplitudes may be plotted against the corresponding periods and this resultant curve is the periodogram. If peaks appear at the suspected periods these may be investigated as regards their reality by statistical tests of significance. Since the seasonal variation introduced a spurious sidereal period, this variation may be subtracted from the periodogram and the remaining portion may be investigated for a true sidereal variation.

One advantage of this method of analysis is that it is possible to obtain the



background of amplitudes. If the background of amplitudes about the periods investigated is of the same magnitude as that of the sidereal periodicity, no valid conclusion can very well be drawn concerning the reality of the suspected

period. In the investigation which is being carried on in connection with the variation in intensity of cosmic rays, it is hoped that this method will give some positive evidence about the sidereal period.
