PROGRESS IN THEORY AND USE OF CONCAVE GRATINGS

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In 1882, Rowland invented the concave grating and thus created a simple and very powerful instrument for obtaining spectra of high resolution and dispersion. Since then, some changes in the mounting of the grating have been introduced, and grazing incidence has been used for exploring short wave-lengths in vacuum spectroscopy.

The investigation of the spectra of polyatomic molecules necessitates the application of the highest possible resolving power over extended regions of the spectrum, excluding thereby interferometers and requiring the extension of the limits in resolution that can be obtained by gratings. A more general treatment of the theory of image formation by the concave grating was therefore started, extending the calculations into three dimensions.

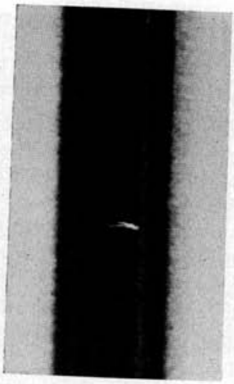
A grating is defined by its radius of curvature, R, and by the spacing of the grooves, d; in addition, by the area on which grooves of length, l, are ruled over a width, w. We can describe then every point P on the grating's surface by Cartesian coordinates l, w and ξ, with origin at the center 0 of the grating. In front of the grating lies a point light source A, with the cylindrical coordinates (origin at 0) r, α and z, and an image point B with r·, β and z·. The coordinates l, z and z· are then parallel.

For the paths of the light diffracted on the grooves the following function holds:

$$F = AP + BP + m \cdot \lambda \cdot \frac{w}{d},$$

where λ is the wavelength and m the spectral order of the light appearing at B. The function F is a complicated power series in R, l, R^{-1}, R^{-2} ....., containing as factors α, β, w and l in ascending powers. To have the light from A focused at B, the function F has to fulfill Fermat's conditions:

$$\frac{\partial F}{\partial w} = 0 \; ; \quad \frac{\partial F}{\partial l} = 0.$$



Mercury line 2537Å, showing the five hyperfinestructure components. The separation of the middle components requires a resolving power of 300,000.

The first condition yields in first approximation the formula for the plane grating, λ as function of α and β, that holds also for the Rowland circle. Further, it defines in higher approximation the Rowland circle as the locus for slit and image. The second condition does not hold for the Rowland circle, but gives rise to astigmatism by drawing each point light source out to a line. But this elongation occurs strictly in the direction of the slit and does not impair the definition of the spectral lines, only their intensity. This astigmatism for all angles α and β is graphically represented in three diagrams, one showing the length z· to which the image of a point is elongated; the second gives the length of the luminous slit necessary to obtain a spectral line that has the maximum intensity over 5 mm along its center; the third graph shows the position in front of the slit that a light source should have in order to be focused on the Rowland circle.

The focusing conditions for Wadsworth's mounting are to be found by setting $r = \infty$ (parallel light). The astigmatism for points outside of the normal has been calculated.

But the partial differential $\frac{\partial F}{\partial w}$ cannot be kept exactly equal to zero. The maximum value allowed is $\frac{\lambda}{4}$, lest extinction of the image occur by spherical aberration. The latter limits the allowed length l and width w of the grating simultaneously, and thus limits the maximum resolving power. A graph represents the allowed areas for all angles α and β.

There are some cross terms contained in the series development of F, involving $w . l^2 . f(\alpha, \beta, R)$. They indicate the presence of coma in the image formation. The magnitude is represented in a graph. In practice it is necessary to limit its amount by restricting the length of the groovés.

The same cross term causes the curvature of the spectral lines, the calculation of which has been carried through. This information is often necessary to insure the highest accuracy in the measurement of the plates, or to utilize the grating as a monochromator with a straight or curved exit slit.

These derivations of the imperfections in the image formation enable one to distinguish between the defects in the adjustment and the errors in the grating itself. They allow a safe quantitative judgment about the maximum resolving power obtainable by a given grating.

In addition they allow one to compensate for errors in the grating by slight changes in the mounting. Thus, the sperical aberration can be utilized to compensate for the error run of the ruling of the grating, by setting not the geometrical center but another point of the grating (along its "equator") tangent to the Rowland circle. Thereby, the exact circle is retained as focal curve. By this means, three big gratings have been adjusted on the same Rowland circle of 30 feet diameter. Each of these gratings yields a resolving power up to 300,000 and 400,000. The advantage of having multiple gratings is that a high intensity can be procured over a wide wavelength range, since each grating has a maximum reflectivity over a small angular range only.

Some reproductions of spectral lines of mercury are shown in the figure, which indicate in the resolved hyperfine structure a resolution of more than 300,000 obtained with our gratings.

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