WHY NOT EXPRESS THE HORSEPOWER IN FOOT-POUNDALS?

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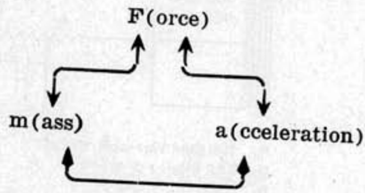
The concepts of mass, acceleration, force, energy, and momentum are the very basis of mechanics, the foundation of all the physical sciences. In spite of the importance of these concepts, teachers in science and engineering must admit that we have a long way to go before we achieve uniformity in the handling of these concepts. The prejudices of workers in physical science in regard to the units used to measure mass, force, energy, and momentum are distressing to the beginning students who must decode the writings and usages of the many authors of physics and engineering texts. It is worthy of note that the students of physics in France, Germany and other European countries escape this confusion because they are familiar with only one system of units—the absolute c. g. s. system. In this country teachers and students must struggle along with both the gravitational and absolute system of units.

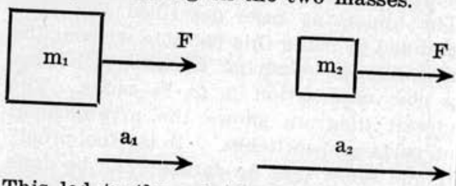
Many teachers of physics feel that the best way to handle the second law equation of Newton is to resort only to absolute units and the equation, F = ma. However, the concept of horsepower is so entrenched in the minds of the engineers and the public in general that they have despaired of entirely dispensing with gravitational units. Many teachers feel that it is necessary to teach the idea of pound of force because of its relation to the horsepower as 550 foot-pounds per second. They feel impelled to devise some system of handling F = ma which will permit solution of problems in either gravitational or absolute units. Many teaching devices have been invented to accomplish this dual purpose.[1] One common method is to make use of two different sets of equations for force, energy, etc., one for use with absolute units, and one for gravitational units.

The concepts of force, mass, kinetic energy, and momentum in reality are not simple concepts. They are due to contributions by philosophical and mathematical thinkers of the caliber of Newton, Galileo, Descartes, Johannes Bernoulli, and Huygens. Ernest Mach[2] in 1888 pointed out that Newton's definitions of mass and force leave us in a logical circle:



These three concepts are all interrelated and physicists have adopted two different procedures to build up a system of units around them.

Procedure I. The relationship between *two different masses* is determined experimentally by the measurement of the *different accelerations* produced by the *same force* acting on the two masses.



This led to the relationship,

$$m_1/m_2 = a_2/a_1 \qquad \text{Equation 1}$$

Eventually the followers of this procedure arbitrarily selected a convenient reference mass, the gram or pound to measure the magnitude of any other mass in terms of this reference mass by the determination of the different accelerations imparted to the two masses by the *same force*, the masses and accelerations being inversely proportional to each other, i. e.,

$$m_2 = m_1(a_1/a_2) — a_1/a_2 \text{ when } m_1 = \text{unity} \qquad \text{Equation 2}$$

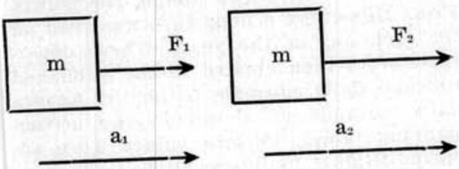
[1] Cf. Perkins, Science, October 14th, 1938; Beardsley, Science 89, No. 2299, p. 58 (1939).
[2] Dampier, William, History of Science, Macmillan Co., New York, 1932, p. 170.

The accelerations of course are expressed in terms of length and time, as cm/sec^2 and ft/sec^2. In this experimental procedure the acting force remains constant. This is implicitly stated in the expression given in Equation 2 since

$$m_1a_1 = m_2a_2 = \text{a constant.} \quad \text{Equation 3}$$

This led to the common statement of the second law of motion, $F = ma$. In this procedure, force must be a derived unit dependent upon the units of mass, length and time. In this way our common absolute units of force, the dyne and poundal came into use.

Procedure II. In this procedure the relationship between *different forces* was determined experimentally by measuring the *different accelerations* imparted by *different forces* on the *same mass*. The accelerations are directly proportional to the applied forces, or



$$F_1/F_2 = a_1/a_2 \quad \text{Equation 4}$$

This method requires the use of a standard force and corresponding acceleration which is usually the weight and acceleration due to the attraction of the earth for the given mass at place where the experimental work is carried out. The last expression is changed to

$$F/Wt = a/g \quad \text{Equation 5}$$

This is sometimes called the ratio form of the second law. This method requires the use of the concept of weight and this naturally led to the gravitational units of force. It should be noted that the concept of mass is not involved in Equation 5. Many physicists contend that Equation 5 in that form is the most fundamental equation in mechanics.[3]

Both procedures have been used which is responsible for the great confusion that exists in regard to the units of mass, force, and energy. "Difficulties . . . . arise from the fact that two systems of measurement of force are actually in use alongside one another, thus producing the appearance of two fundamentally different definitions of force."[4] Unfortunately, the followers of Procedure II adopted the concept of pound of force. In order to get the notion of pound of force, it was necessary to use the pound of mass. To add to the confusion a new unit of mass was invented, the slug. In order to have the second law in the form $F = ma$ mathematically, Equation 5 is slightly altered,

$$F = Wt(a/g) = W/g(a) = Ma \quad \text{Equation 6}$$

When F is expressed in pounds of force and a in ft/sec^2, then the new unit of mass, the slug, is a derived unit and is defined as that mass in which a force of one pound will produce an acceleration of 1 ft/sec^2. In effect the followers of Procedure II in a round about manner go from our unit of mass, the pound, to the pound of force and back to the hypothetical unit of mass, the slug, in order to use the second law in the form, $F = Ma = Wt/g(a)$. In practice this results in the use of two sets of equations in mechanics as illustrated below:

Results of Procedure I

$P.\,E. = mgh$
$K.\,E. = \frac{1}{2}\,mv^2$
$F = mv^2/r$

Procedure II

$P.\,E. = mgh/g = mh$
$K.\,E. = \frac{1}{2}\,mv^2/g$
$F = mv^2/rg$, etc.

Often the followers of Procedure II do not actually stress the use of the term slug.

As stated before the reluctance of many teachers to discard the gravitational units of force is due to the use of the foot-pound in the horsepower unit. But it is still possible to retain the horsepower unit and express in absolute units as *foot-poundals per second*. For some reason this is not done by writers of physics and engineering texts. The legal definition of the pound in this country as stated by the Bureau of Standards is a mass equal to 453.5920 grams. The average value of g at sea level and 45 degrees latitude is 32.1740 ft/sec^2. Hence, the horsepower in absolute units will be,

$$P = W/t = F\,xS/t = mgS/t = \frac{(550\text{ lbs})\,(32.1740\text{ ft/sec}^2)\,(1\text{ ft})}{1\text{ sec}}$$

$$= 17696 \text{ foot-poundals per second}$$

For ordinary calculations, the horsepower can be expressed as 17,700 foot-

[3] Cf. Huntington, Science, *41*, 207-209 (1915).
[4] Bavink, The Natural Sciences, The Appleton-Century Co., New York, 1932.

poundals per second with an error of 0.02 per cent.

For problems involving the conversion of mechanical energy into heat, the mechanical equivalent of heat can likewise be expressed in foot-poundals per B. T. U.

$$1 \text{ Calorie} = 4.184 \text{ joules}$$
$$1 \text{ B. T. U.} = 252 \text{ cal} = 252 \times 4.184 \text{ joules}$$
$$= 2.50 \times 10^4 \text{ foot-poundals}$$

Expressing the horsepower and the mechanical equivalent of heat in absolute English units means that only one set of equations is needed to measure force, work, potential and kinetic energy, etc. The parallelism between the absolute metric and English units is quickly grasped by the beginning student in physics.