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SERIES RESONANCE IN AN R-L-C CIRCUIT

RICHARD L. KNUTH
St. Procopius College, Lisle

In the electronics and electricity textbooks which we use in college, resonance curves may be found using high frequencies such as a million and a half cycles per second. I thought it would be interesting to study resonance phenomena at audio frequencies, in the neighborhood of 5 to 10 kilocycles per second. At these frequencies it can be shown by impressing it on the oscilloscope that the input voltage has a pure sinusoidal waveform. This cannot be done at the very high frequencies.

An electronic oscillator driving a power amplifier was used to produce a constant signal input voltage with variable frequency. This voltage was then fed into the circuit (fig. 1).

A suitable coil was wound and its inductance found to be 1.15 millihenries. It was then placed in series with a condenser, and the generator voltage was applied and kept constant. In such a circuit, the current is dependent on the frequency of the generator. Thus begins the study of series resonance (fig. 2). This is a graph of the current in the circuit plotted against the frequency of the generator. According to theory, resonance occurs at $F = \dfrac{1}{2\pi\sqrt{LC}}$. Using the coil of 1.15 millihenries and a condenser of ¼ microfarad, the resonant frequency is 10,250 cycles per second. Now if we change capacitance, the resonance point will shift either to the right or to the left as we decrease or increase the capacitance.

There are several things which could be studied in detail in relation to series resonance, for example, sharpness of resonance, resonance of voltage across the coil, or resonance of voltage across the condenser. In this case, the two points of resonance vary from the point of current resonance by about a hundred cycles per second.

However, I thought that it would be of particular interest to study the variation of resistance in the coil as the frequency is increased. We know that the resistance of a wire increases with the frequency. This is due to the "skin effect" (the current flowing through the wire is concentrated in the outer layers). Since the current at resonance is given by I = E/R, where *E* is the effective input voltage and *I* is the effective current in the circuit, *R*, the resistance of the coil, can be found. By changing condensers several times, we get several points of resonance,

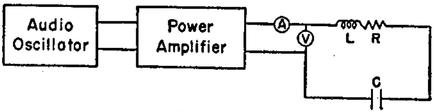

FIG. 1.

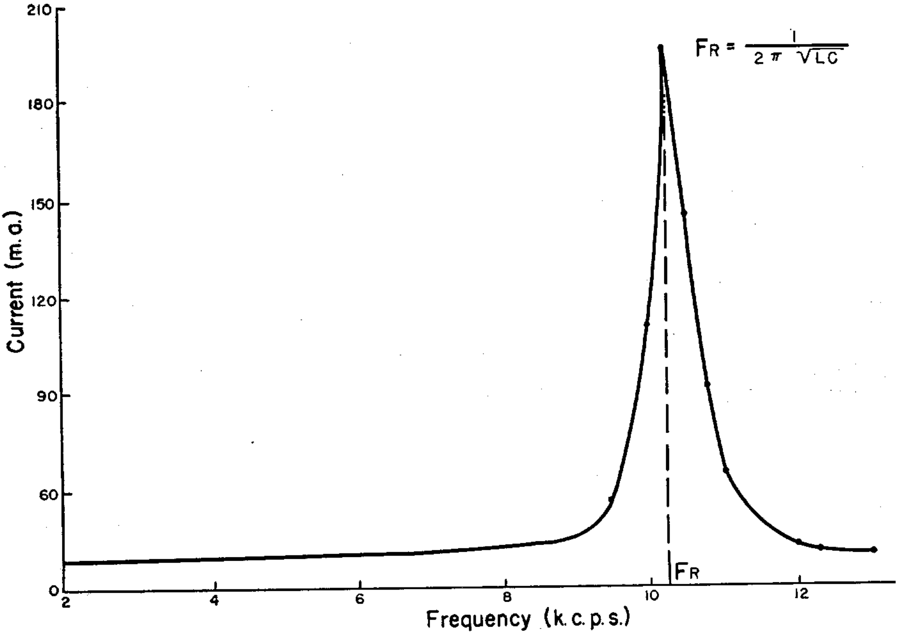

FIG. 2.

and computing the resistance at each point, we can plot a graph of resistance against frequency (fig. 3).

A very important quantity encountered in the study of resonance phenomena is the figure of merit of the coil. It is defined as $\frac{2\pi FL}{R}$, the ratio of the reactance of the coil to its resistance, and is symbolized by Q. In theory it can be shown that the voltage across the coil or condenser, at resonance, is Q times as large as the input voltage. At the resonant frequency given before, my coil had a Q of 40. But the Q does not remain constant. Since both the numerator and denominator of Q increase with frequency, the Q may either increase or decrease. For my particular coil and in the frequency range of 2 to 10 kilocycles per second, the Q is always increasing.

Resonance phenomena in coupled circuits and parallel circuits could be similarly studied. The most interesting is resonance in coupled circuits. However, I chose to describe series resonance because it is the simplest and yet illustrates how experiments of this kind can be done.

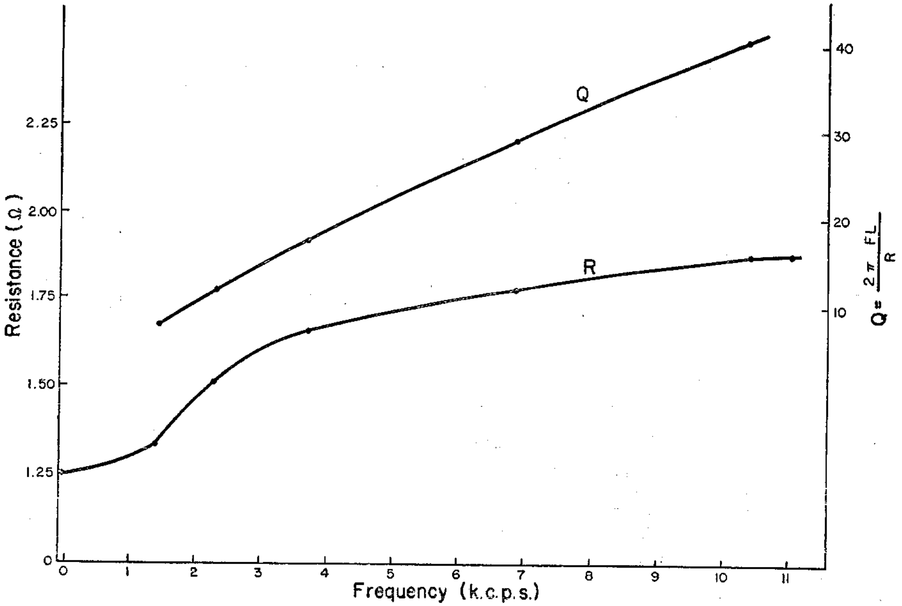
Q
R
Resistance (Ω)
2.25
2.00
1.75
1.50
1.25
0
0
1
2
3
4
5
6
7
8
9
10
11
Frequency (k.c.p.s.)
40
30
20
10
$Q = \frac{2\pi FL}{R}$

FIG. 3.