APPLICATIONS OF THE STATISTICAL SLOPE ORIENTATION DIAGRAM

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Most analytical methods dealing with areal slope relations are concerned primarily with but one of the two slope elements, namely, slope angle. The other element, slope direction or slope strike, which in many cases may be as significant as slope angle, is not considered.

The purpose of this paper is to point out a system whereby quantitative slope studies may be undertaken which involve slope direction relations as well as those of slope angles.

The successful application of this scheme to topographic problems is dependent upon the verity of the following concept. All topographic surfaces are a function of numerous factors such as climate, nature of underlying rock material, age of surface, geographical position, and geological processes involved. Each land surface has formed as a result of some combination of these factors, and each factor is in part responsible for the complicated slope pattern produced. If it were possible to simplify the slope picture and blend it into a clear-cut image of quantitative three-dimensional relations, we might have an effective tool for determining certain of the numerous factors which have played significant roles in fashioning a land surface.

The writer believes that we have such a means of representing the various types of slope relationships in the simple and lucid delineation which is termed the "statistical slope orientation diagram." This diagram, briefly called the SSO diagram, may be prepared from the topographic map of an area. Slope directions and slope angles are measured at a large number of uniformly distributed points on the map. The number of measurements should be sufficient to give a statistical coverage; and this number will vary with the texture, grain, homogeneity, etc., of the topography. Normally a few hundred points are sufficient to represent a hundred square miles, although in some instances fewer points may be adequate and in others many more may be required. Having made the necessary number of slope measurements, the next problem is to plot them all on a common diagram. This may be done by the projection method.

Each slope surface is visualized as a plane cutting the center of a sphere (the projection sphere). A line constructed perpendicular to this plane and passing through the sphere's center is called the pole of the plane. This pole will intersect the sphere at two points, one generally on the upper hemisphere the other on the lower hemisphere. Only the upper point need be considered here. This point may be said to represent the plane in spherical projection. Since this projection involves a three-dimensional model, a more conven-

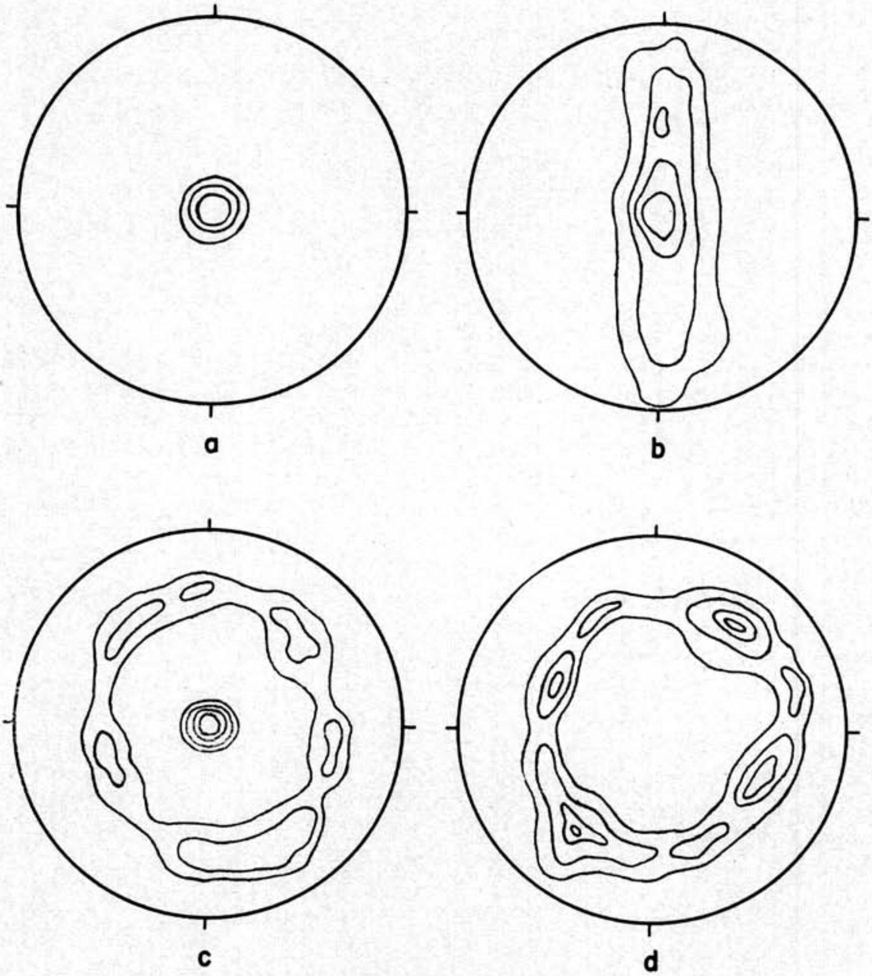

FIG. 1.—Hypothetical statistical slope orientation diagrams.

ient method is to use a two-dimensional plot as is employed in crystal projection, petrofabric diagrams, and map making.

For our present purpose an equal-area projection net (commonly called a Schmidt net or equi-areal equatorial projection net) is used. The slope angle and slope direction at each point on the topographic map are represented by a single point on the projection net just as in the case of spherical projection mentioned above. The point on the plot represents the pole of the slope plane, the pole being the line normal to the slope plane. When all slope values are plotted the pole diagram is completed (see fig. 2a). To render these data still more intelligible the density distribution of poles may be shown by contour lines in the same manner employed in petrofabric diagrams (see fig. 2b). Only the central part of the equal-area projection need to be shown as few points fall outside the 30° circle. Details for the construction of these equal-area diagrams may be found in the standard works of Knopf and Ingerson (1) and Fairbairn (2).

Interpretation of SSO Diagrams

In order to understand the relationship between the SSO diagram and topography, a few examples will be considered. The SSO diagram of a nearly flat plain would show a strong concentration of poles at the center. This concentration is called a maximum and is illustrated by the hypothetical example shown in figure 1a. A very smooth but somewhat sloping surface would show a maximum slightly off-center in the SSO diagram. The degree of off-centering would be determined by the slope angle of the surface and the direction of off-centering would be determined by the direction of slope. The diameter of the central maximum concentration decreases as the topographic plain approaches a geometrical plane. The shape of the maximum changes from roughly circular, for areas with no preferred stream orientation, to highly elliptical or elongate for areas of marked stream parallelism.

Figure 1b shows a concentration of poles along a north-south line of the diagram. By analogy with petrofabric diagrams this elongate or beltlike concentration is here called an incomplete girdle. The north-south girdle represents a concentration of topographic slopes which trend in an east-west direction; and, therefore, the valleys and ridges trend east-west. In other words, incomplete girdles are formed perpendicular to valley trends, and figure 1b is a hypothetical SSO diagram of simple ridge-and-valley topography in which there is a marked east-west parallelism of streams. From this diagram we may assume that valley cross sections are essentially symmetrical because the distribution of poles to the north and south of the center point is symmetrical. If north slopes, for example, had been somewhat steeper than south slopes, the incomplete girdle would have extended farther to the north of the center point than to the south of it.

In areas which show two directions of preferred stream orientation, such as is characteristic of trellis drainage, two elliptical bands or incomplete girdles would intersect (at approximately 90°) at the mid-point of the SSO diagram. The stream set associated with the steeper slopes would be indicated by the more extensive incomplete girdle.

Another type of diagram is that characteristic of the canyon-and-mesa country (see fig. 1c). Here the slopes fall into two highly contrasting groups: the very steep and the very gentle. In general the nearly flat surfaces cover the greater map area; and a very strong concentration of poles, therefore, will show up at the center of the SSO diagram. Out beyond this strong central maximum a ring of low pole-concentration will occur. This in turn will be surrounded by a more or less continuous ring or small-circle girdle of m o d e r a t e pole-concentration which represents the steep canyon

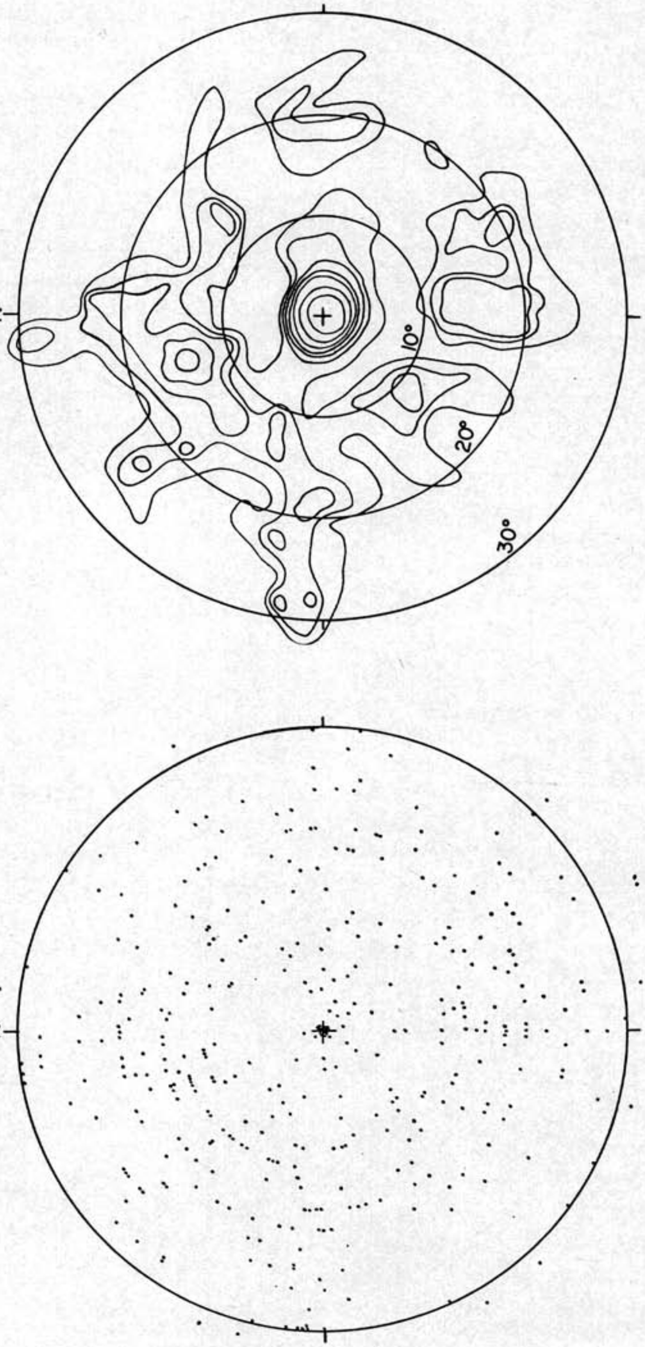

FIG. 2 a.—Pole diagram of the Arnoldsburg quadrangle, West Virginia. 392 poles; outer circle = 30°. b.—SSO diagram of the Arnoldsburg quadrangle, West Virginia. 392 poles; contours 1, 1.5, 2, 3, 4, 6, 10%; outer circle = 30°.

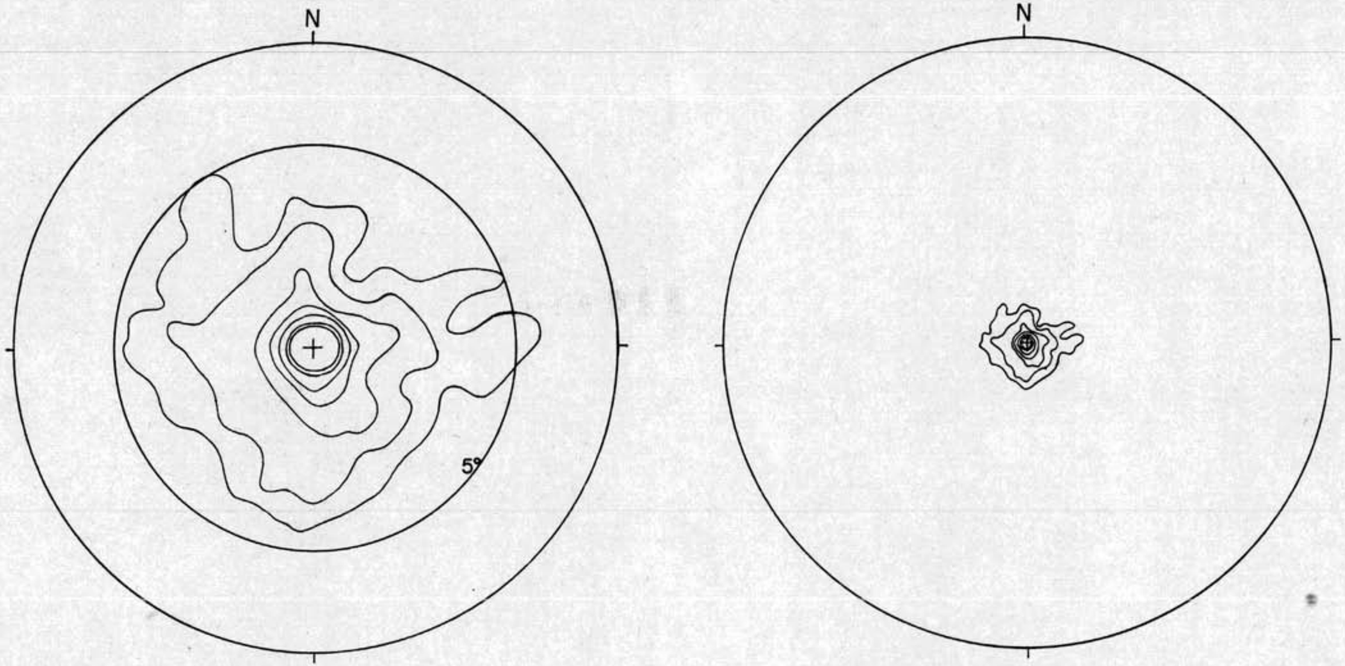

FIG. 3 a.—SSO diagram of the Ellenton quadrangle, South Carolina—Georgia. 535 poles; contours 1, 2, 4, 6, 12, 18, 24%; outer circle = 7½°. b.—Same as figure 3a but reduced four times. Outer circle = 30°.

slopes. Any preferred stream orientation will be revealed in this diagram by the symmetry of the outer annular concentration. Poles uniformly distributed throughout this small-circle girdle indicate no preferred orientation of streams, whereas broken rings or incomplete small-circle girdles show a preferred slope orientation which may be due to the orientation of streams or the control of rock structure.

In most of the SSO diagrams studied so far by the writer, a high pole-concentration appears at the center indicating the high percentage of area underlain by gentle slopes in many types of topography. In striking contrast to this apparently general tendency is the badland topography. Here steep slopes predominate and a pronounced small-circle girdle is expected with a low or inconspicuous maximum at the center of the SSO diagram (see fig. 1d). Again, as in the canyon-and-mesa country, any structural control or preferred stream orientation may be reflected in the nature of the small-circle girdle.

Examples of SSO Diagrams

Consideration may now be given to actual examples of SSO diagrams for contrasting types of topography. Figure 2b is an SSO diagram of the Arnoldsburg quadrangle, W. Va. The area represents a mature topography with dendritic drainage and an average relief of about 400 feet. The map used for constructing the diagram was the edition of 1907, the area having been surveyed in 1905 on a scale of 1:62,500 and a contour interval of 20 feet.

The uniformity of topographic elements throughout the area seems to suggest that the 392 poles chosen for the diagram are adequate to represent statistically the numerous slope relationships. In the diagram very few poles fall outside the 30° circle indicating that slopes greater than 30° are relatively rare in the Arnoldsburg quadrangle. For convenience the 10°, 20°, and 30° circles are shown on the SSO diagram. Study of this diagram reveals the fact that slope angles fall into two groups: those of low dip concentrated within the 5° circle and those falling between about 10° and 20°. Slopes between about 5° and 10° are conspicuously absent and only local concentrations of poles beyond the 20° circle are to be noted. There appears to be, therefore, a strongly developed maximum at the center and numerous weaker maxima radially arranged at about the 15° circle. This latter group of maxima constitutes a somewhat interrupted small-circle girdle. The SSO diagram brings out very clearly a tendency for gentle slopes (less than 5°) to pass rather abruptly into intermediate slopes (10°-20°) and from these gradually into steep slopes (greater than 20°).

A rather pronounced incomplete girdle extends in an east-west direction due to the high frequency of east and west slopes which incline at about nearly all angles up to about 32°. A second incomplete girdle trends at about right angles to the first and reflects a slightly weaker concentration of north and south slopes. A faint and somewhat questionable incomplete girdle trends NW-SE.

The average value for the gentle slopes (those less than 8°) is not zero degrees but a low angle to the east. This is indicated by the somewhat non-concentrically arranged contour lines which appear more widely spaced on the east side of the central maximum than on the west. In contrast to this, the steep slopes are more numerous in the northwest quadrant, whereas the intermediate slopes appear somewhat more uniformly distributed. Considering the diagram as a whole, it is approximately bilaterally symmetrical but it appears to be tilted off-center slightly in a northwest direction. The symmetry of this SSO diagram may be considered monoclinic. It should be noted, furthermore, that the symmetry plane trends roughly parallel to the principal drainage line of the areas and the northwesterly tilt of the diagram is in the same direction as that of the drainage. A study of the topographic map may suggest to the reader other relationships brought out by the SSO diagram of this quadrangle.

A region of low relief and youthful topography will now be considered for purposes of comparison. The SSO diagram shown in figure 3 was prepared for the Ellenton quadrangle, S. C.-Ga. This quadrangle was surveyed in 1918-1919 on a scale of 1: 62,500 and contour interval of 10 feet. The diagram is a plot of 535 poles and does not include the flat swampy area along the Savannah River. This greater number of poles, covering slightly less than a quadrangle, gives a somewhat denser sample than that for the Arnoldsburg quadrangle. After examining the topographic map, the reader will understand why a greater number of measurements is required for this less homogeneous topography.

The SSO diagram of the Ellenton quadrangle was first prepared on a scale four times that of the Arnoldsburg quadrangle in order to bring out the finer details of pole distribution. This diagram is shown in figure 3a and it should be noted that the outer circle represents 7½°. The diagram was reduced, for purpose of comparison, later to the same scale as that of the Arnoldsburg quadrangle, and it is shown in figure 3b in its proper relation to the 30° circle. Perhaps the most striking difference between the diagrams of these two quadrangles is that in the Ellenton quadrangle nearly all poles lie within the 5° circle and the area occupied by the central maximum is extremely small (about 1° radius). The diagram appears roughly radially symmetrical but the enlargement (4 times) brings out the asymmetrical character more clearly. The central maximum is very well centered with a pole concentration of 29% at the very center. This maximum indicates the high percentage of land surface under a slope of one degree. For slightly greater slopes (1°-2°) the contours become less concentric and more off-center in a south-east direction. The steepest slopes (2°-5°) are somewhat more abundant toward the south and west, as is indicated by the direction and amount of off-centering of the one percent and two percent contour lines.

Two embryonic incomplete girdles may be detected in the SSO diagram, one trending approximately east-west the other approximately northwest-southeast. The steepest of all slopes have their poles at the ends of these incomplete girdles. In a youthful topography the steepest slopes are generally confined to the walls of narrow valleys which are being cut by the most active streams. The distribution of the steepest slopes on the SSO diagram, therefore, should reveal the stream pattern of the region. In the SSO diagram for the Ellenton quadrangle, the two embryonic incomplete girdles are not sharply separated from each other but are somewhat gradational. They indicate valley trends in north-south and northeast-southwest directions. Upon checking the topographic map, one finds that the larger valleys do trend in these principal directions; and the fact that these valley directions are dendritically related explains why the incomplete girdles are somewhat gradational.

Conclusion

An attempt has been made to show that numerous slope relations may be brought out by the use of statistical slope orientation diagrams. These diagrams offer us an entirely new means of comparing land surfaces not presented by other analytical methods. Some of the errors likely to be involved in this analytical scheme include those of sampling, slope measurement, diagram preparation and map errors. In spite of these, however, the method is believed to be of definite value. The general application of the SSO diagrams is apparent but further study is required in order to determine the limitations of the method. It is suggested that the diagrams may be of considerable assistance in solving a variety of geomorphic problems such as: differentiation of till sheets; control of jointing, lithology and rock structure upon topography; and the relative modification of topography by glaciation.

It has not been possible in this brief paper to consider in detail the method of sampling topography, the preparation of slope diagrams, and the numerous problems relative to the use and interpretation of SSO diagrams. These topics are being investigated, and it is expected that the findings will be presented at a future time.

REFERENCES

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2. Fairbairn, H. W. (1949) Structural petrology of deformed rocks, Addison-Wesley, Cambridge, Mass.